ECE 121
Electronics (1)

Lec. 5: BJT Modeling and re Transistor Model (small signal analysis)

Instructor

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Agenda

- Amplification in the AC Domain
- BJT transistor Modeling
- The r_e Transistor Model (small signal analysis)

Amplification in the AC Domain

Amplification in the AC Domain

$\eta = P_o/P_i$ cannot be greater than 1.

In fact, a *conversion efficiency* is defined by $\eta = P_{o(ac)}/P_{i(dc)}$, where $P_{o(ac)}$ is the ac power to the load and $P_{i(dc)}$ is the dc power supplied.

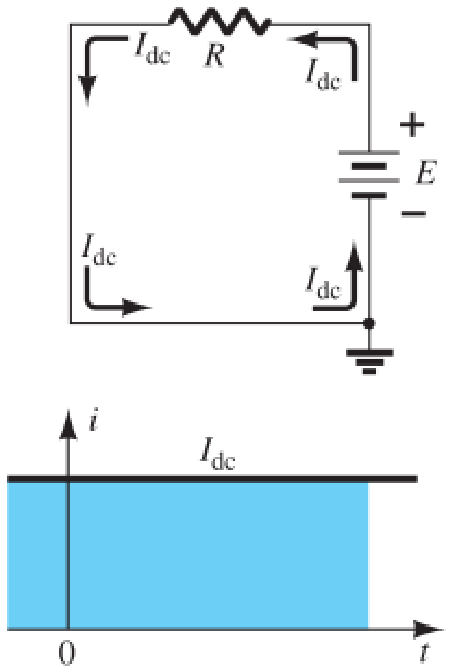


FIG. 5.1
Steady current established by a dc supply.

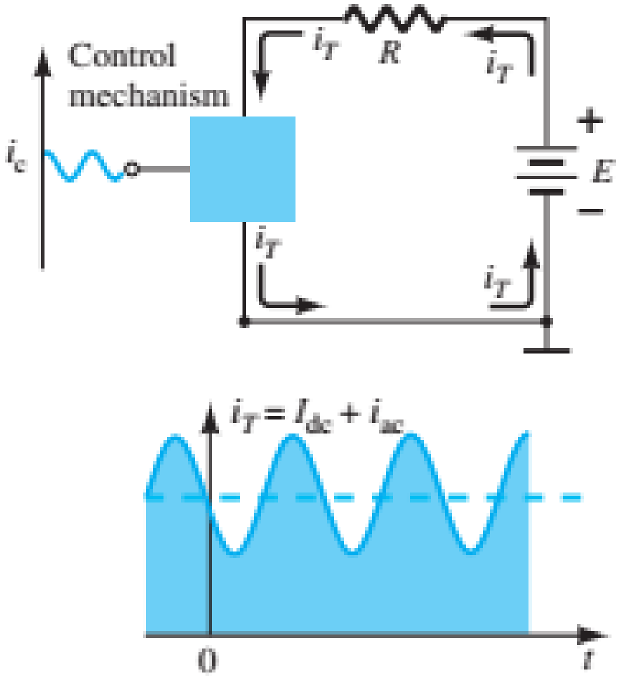
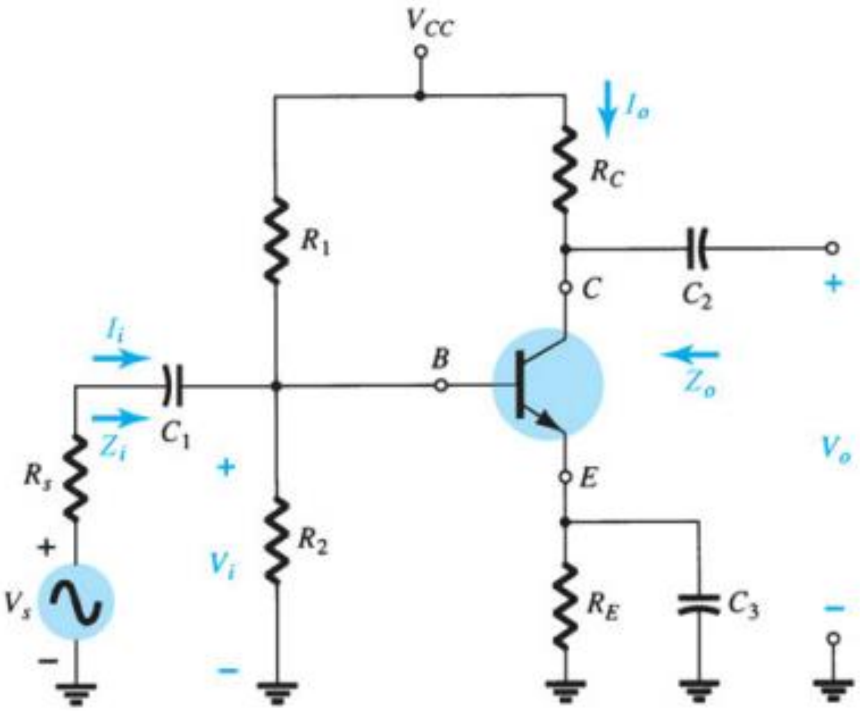


FIG. 5.2
Effect of a control element on the steady-state flow of the electrical system of Fig. 5.1.

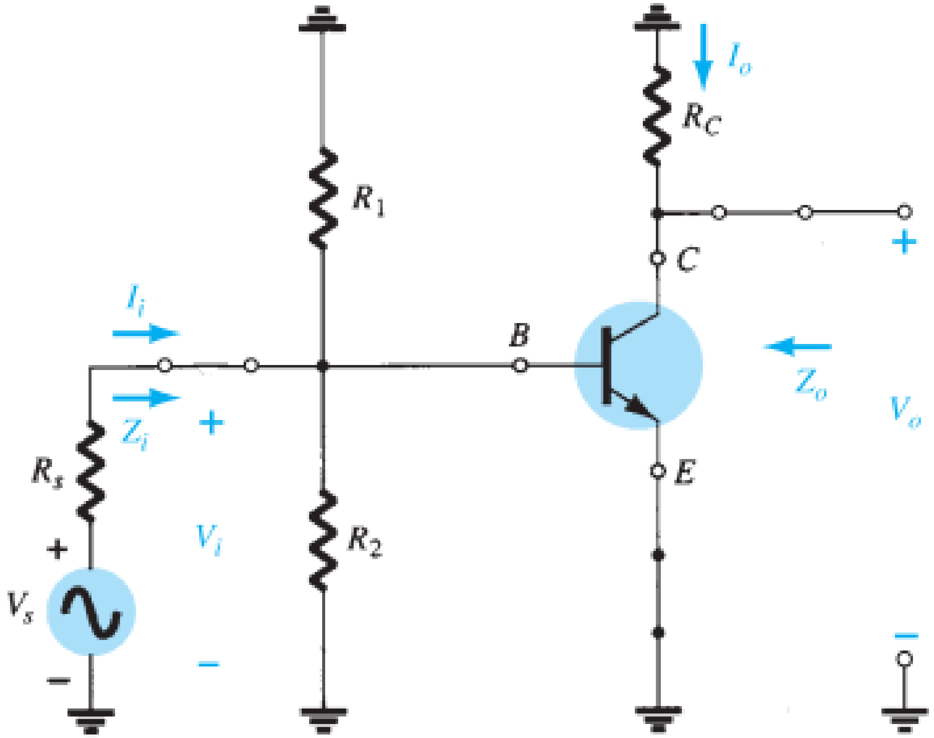
The superposition theorem is applicable for the analysis and design of the DC and AC components of a BJT network, permitting the separation of the analysis of the dc and ac responses of the system.

BJT Transistor Modeling

BJT Transistor Modeling

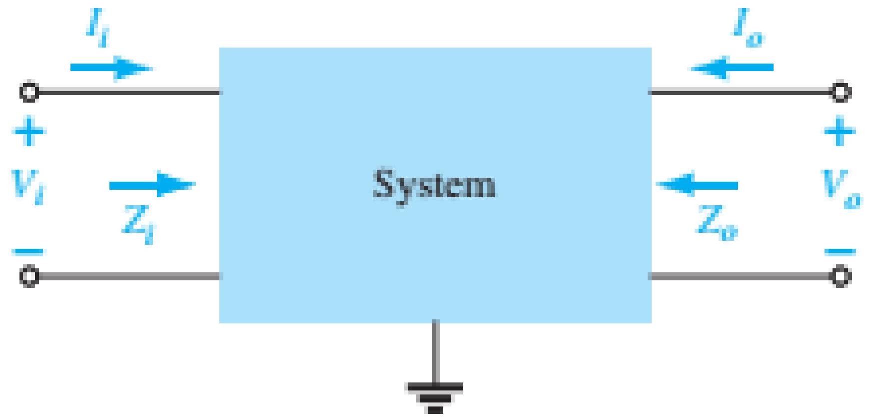


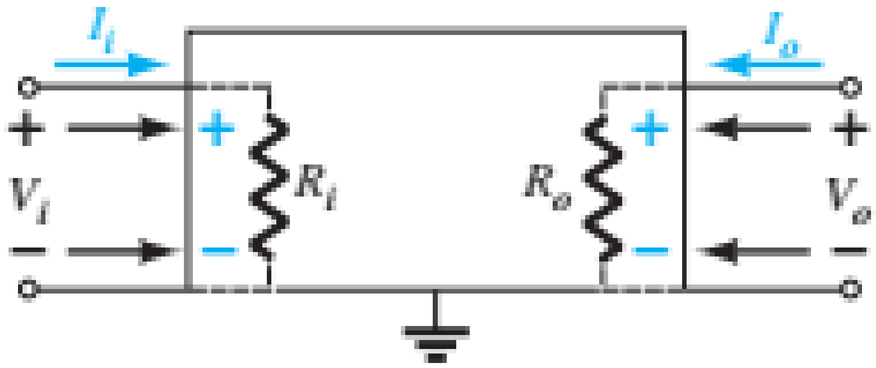
Ac analysis



BJT Transistor Modeling (1 of 2)

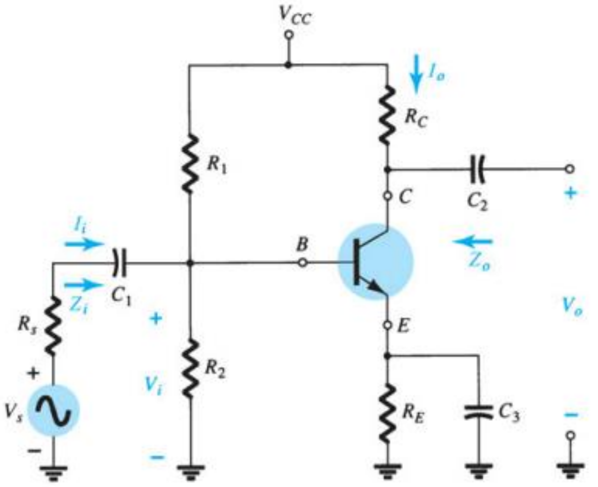
- A model is a combination of circuit elements, properly chosen, that best approximates the actual behavior of a semiconductor device under specific operating conditions.
- Any electronic system has some important parameters have to be determined
  - Input and Output Voltage
  - Input and Output Impedance
  - Input and Output Current

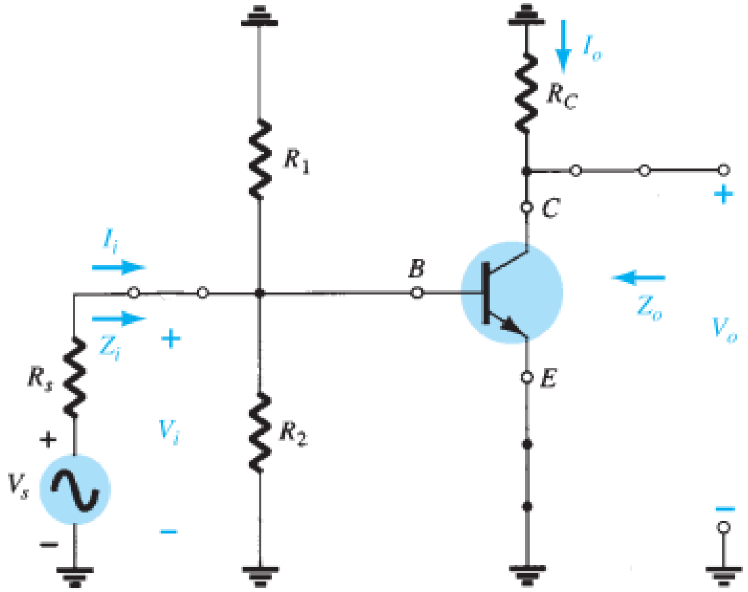


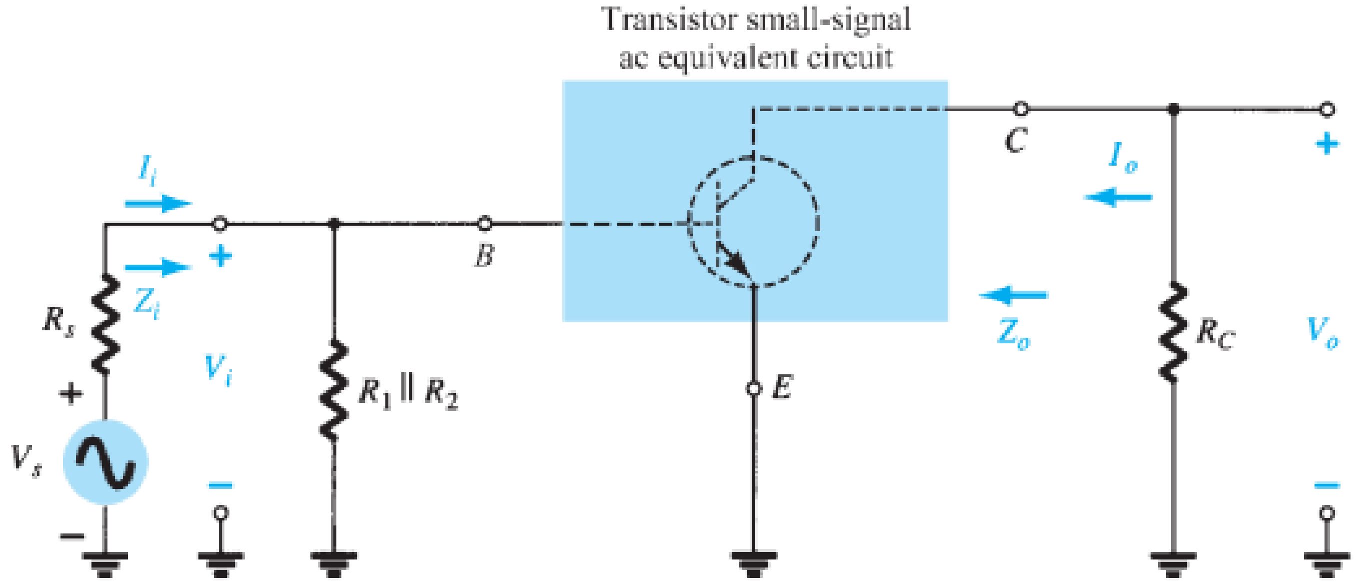


BJT Transistor Modeling (2 of 2)

- The ac equivalent of a transistor network is obtained by:
  1. Setting all dc sources to zero and replacing them by a short-circuit equivalent
  2. Replacing all capacitors by a short-circuit equivalent
  3. Removing all elements bypassed by the short-circuit equivalents introduced by steps 1 and 2
  4. Redrawing the network in a more convenient and logical form







The r_e Transistor Model

- Common Emitter Configuration
- Common Base Configuration
- Common Collector Configuration
- r_e Model in Different Bias Circuits

The r_e Transistor Model (CE)

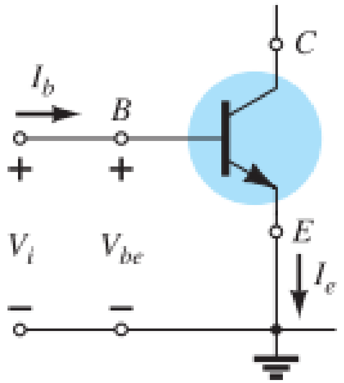


FIG. 5.8
Finding the input equivalent circuit for a BJT transistor.

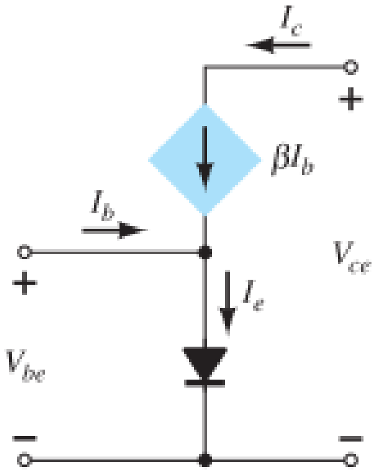
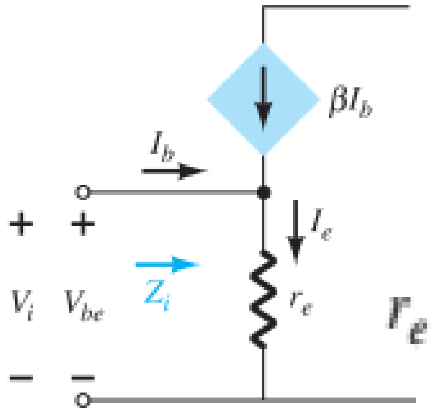


FIG. 5.12
BJT equivalent circuit.



$$r_e = 26\,\text{mV}/I_E$$

FIG. 5.13
Defining the level of Z_i.

$$Z_i = \frac{V_i}{I_b} = \frac{V_{be}}{I_b}$$

$$V_{be} = I_e r_e = (I_c + I_b) r_e = (\beta I_b + I_b) r_e$$

$$= (\beta + 1) I_b r_e$$

$$Z_i = \frac{V_{be}}{I_b} = \frac{(\beta + 1) I_b r_e}{I_b}$$

$$\boxed{Z_i = (\beta + 1) r_e \cong \beta r_e}$$

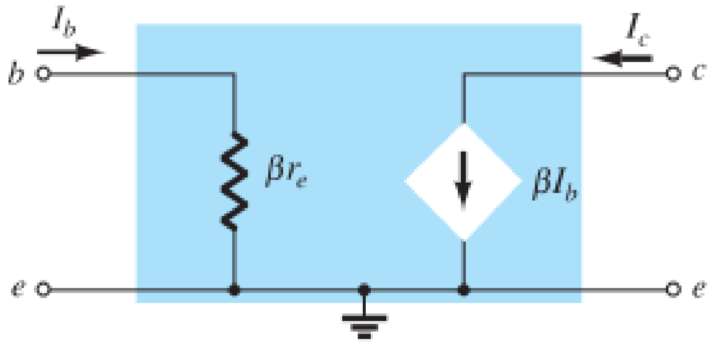


FIG. 5.14
Improved BJT equivalent circuit.

The r_e Transistor Model (CE)

Early Voltage

$$\text{Slope} = \frac{\Delta y}{\Delta x} = \frac{\Delta I_C}{\Delta V_{CE}} = \frac{1}{r_o}$$

$$r_o = \frac{\Delta V_{CE}}{\Delta I_C}$$

$$r_o = \frac{\Delta V}{\Delta I} = \frac{V_A + V_{CE_Q}}{I_{C_Q}}$$

$$r_o \cong \frac{V_A}{I_{C_Q}}$$

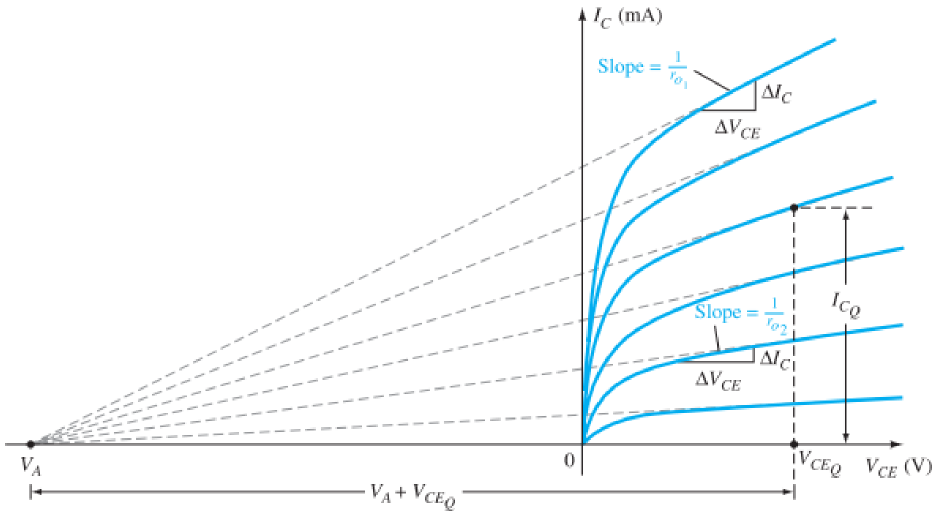


FIG. 5.15
Defining the Early voltage and the output impedance of a transistor.

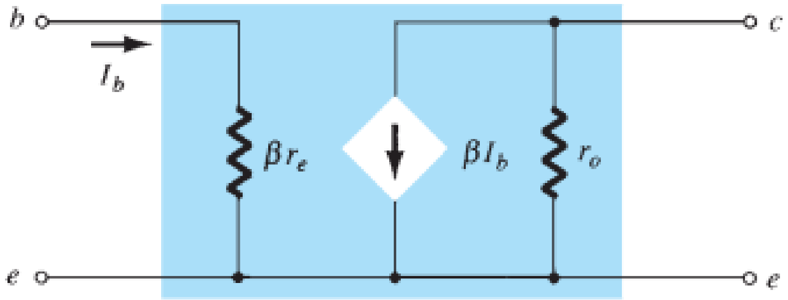


FIG. 5.16
r_e model for the common-emitter transistor configuration including effects of r_o.

The r_e Transistor Model (CB)

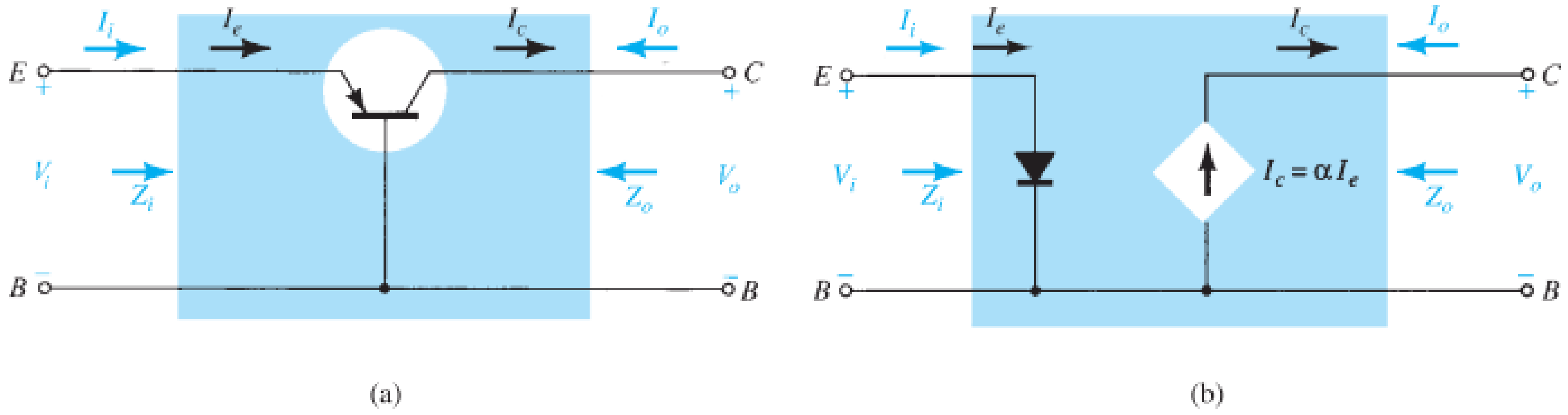


FIG. 5.17
(a) Common-base BJT transistor; (b) equivalent circuit for configuration of (a).

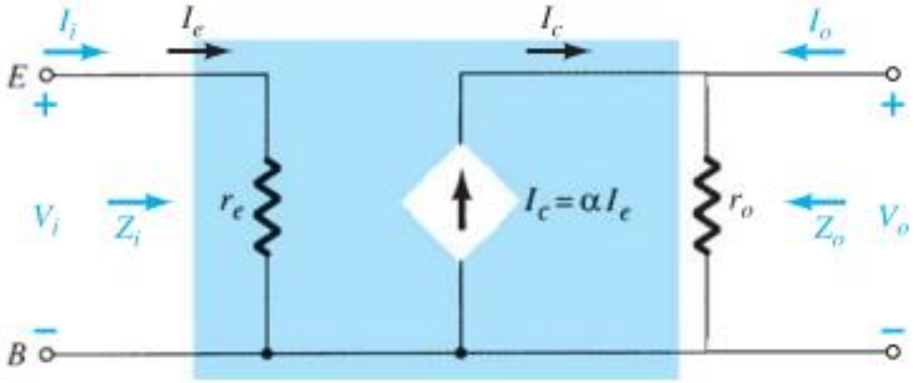


FIG. 5.18
Common base r_e equivalent circuit.

The r_e Transistor Model (CC)

- For the common-collector configuration, the model defined for the common-emitter configuration of is normally applied rather than defining a model for the common-collector configuration.

npn versus pnp

- The dc analysis of *npn* and *pnp* configurations is quite different in the sense that the currents will have opposite directions and the voltages opposite polarities.
- However, for an ac analysis where the signal will progress between positive and negative values, the ac equivalent circuit will be the same.

C.E. Fixed Bias Configuration

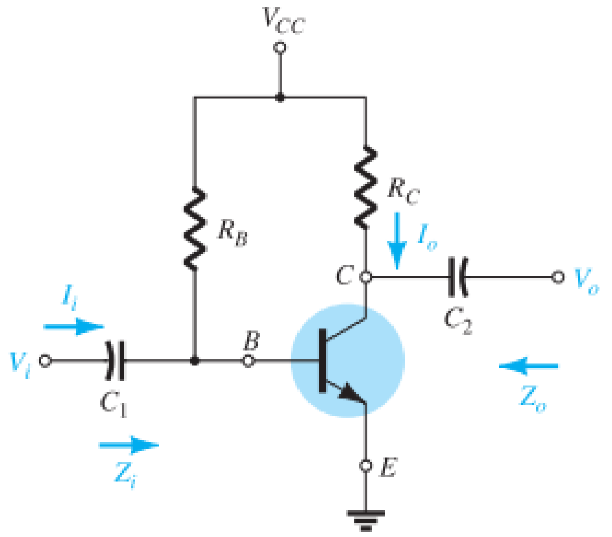


FIG. 5.20
Common-emitter fixed-bias configuration.

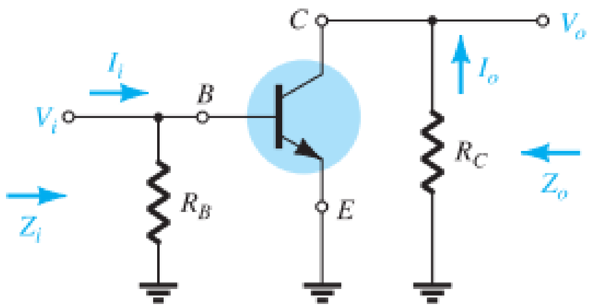


FIG. 5.21
Network of Fig. 5.20 following the removal of the effects of V_{CC}, C_1, and C_2.

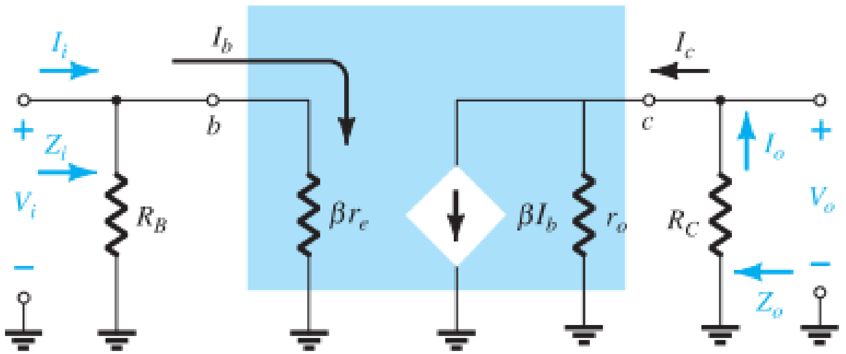


FIG. 5.22
Substituting the r_e model into the network of Fig. 5.21.

$$Z_i = R_B \| \beta r_e \quad \text{ohms}$$

$$Z_i \cong \beta r_e \Big|_{R_B \geq 10\beta r_e} \quad \text{ohms}$$

$$Z_o = R_C \| r_o \quad \text{ohms}$$

$$Z_o \cong R_C \Big|_{r_o \geq 10 R_C}$$

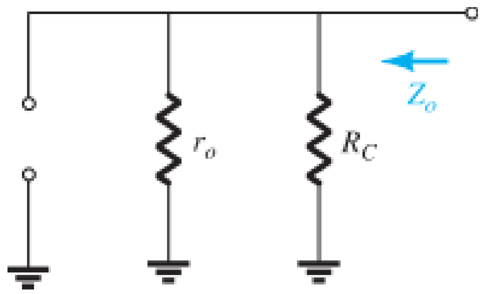


FIG. 5.23
Determining Z_o for the network of Fig. 5.22.

$$V_o = -\beta I_b (R_C \| r_o)$$

$$I_b = \frac{V_i}{\beta r_e}$$

$$V_o = -\beta\left(\frac{V_i}{\beta r_e}\right)(R_C \| r_o)$$

$$A_v = \frac{V_o}{V_i} = -\frac{(R_C \| r_o)}{r_e}$$

$$A_v = -\frac{R_C}{r_e} \Big|_{r_o \geq 10 R_C}$$

C.E. Fixed Bias Configuration (Phase relationship)

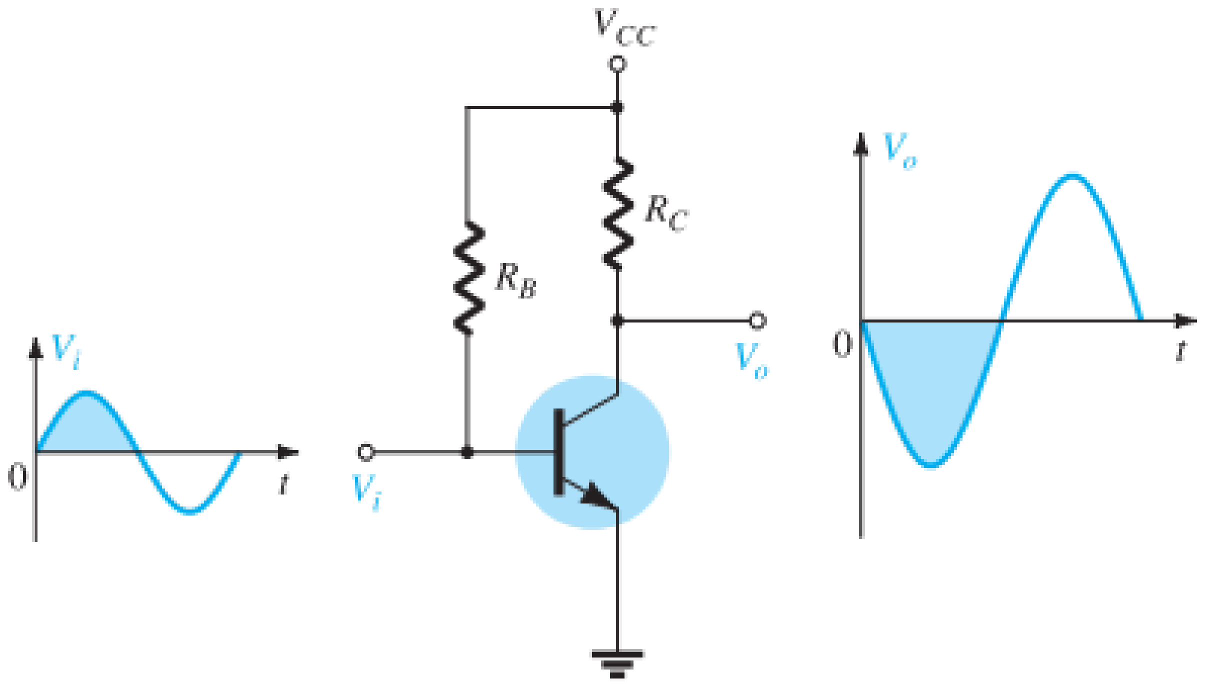


FIG. 5.24
Demonstrating the 180° phase shift between input and output waveforms.

C.E. Fixed Bias Configuration (Example)

EXAMPLE 5.1 For the network of Fig. 5.25:

a. Determine r_e.
b. Find Z_i (with $r_o = \infty\,\Omega$).
c. Calculate Z_o (with $r_o = \infty\,\Omega$).
d. Determine A_v (with $r_o = \infty\,\Omega$).
e. Repeat parts (c) and (d) including $r_o = 50\text{ k}\Omega$ in all calculations and compare results.

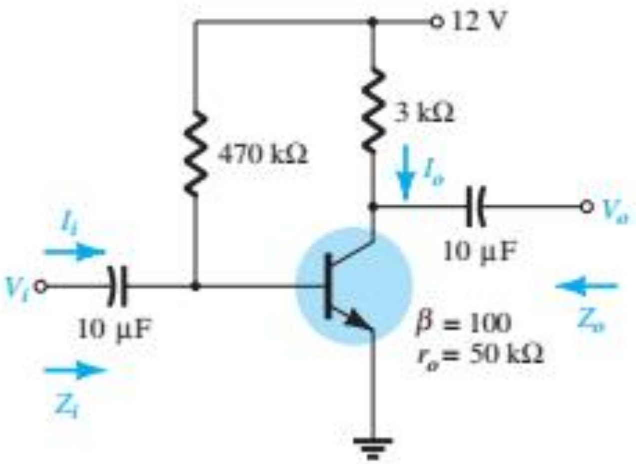


FIG. 5.25
Example 5.1.

Solution:

a. DC analysis:

$$I_B = \frac{V_{CC} - V_{BE}}{R_B} = \frac{12\text{ V} - 0.7\text{ V}}{470\text{ k}\Omega} = 24.04\ \mu\text{A}$$

$$I_E = (\beta + 1)I_B = (101)(24.04\ \mu\text{A}) = 2.428\text{ mA}$$

$$r_e = \frac{26\text{ mV}}{I_E} = \frac{26\text{ mV}}{2.428\text{ mA}} = \mathbf{10.71\ \Omega}$$

b. $\beta r_e = (100)(10.71\ \Omega) = 1.071\text{ k}\Omega$

$$Z_i = R_B \| \beta r_e = 470\text{ k}\Omega \| 1.071\text{ k}\Omega = \mathbf{1.07\text{ k}\Omega}$$

c. $Z_o = R_C = \mathbf{3\text{ k}\Omega}$

d. $A_v = -\dfrac{R_C}{r_e} = -\dfrac{3\text{ k}\Omega}{10.71\ \Omega} = \mathbf{-280.11}$

e. $Z_o = r_o \| R_C = 50\text{ k}\Omega \| 3\text{ k}\Omega = \mathbf{2.83\text{ k}\Omega}$ vs. $3\text{ k}\Omega$

$$A_v = -\frac{r_o \| R_C}{r_e} = \frac{2.83\text{ k}\Omega}{10.71\ \Omega} = \mathbf{-264.24} \text{ vs. } -280.11$$

C.E. Voltage-Divider Bias

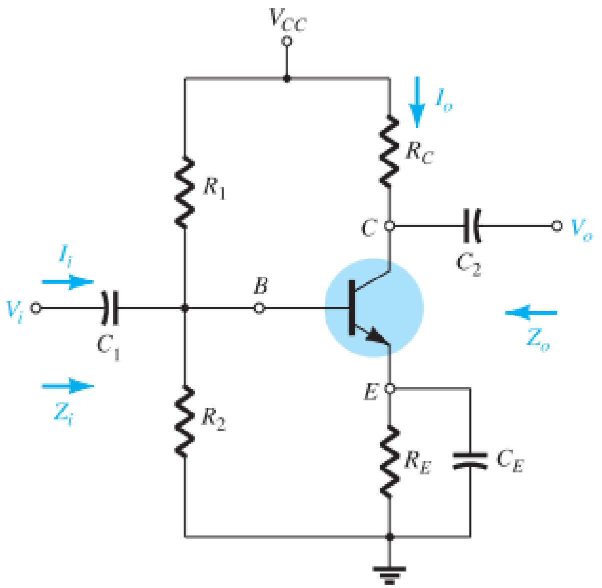


FIG. 5.26
Voltage-divider bias configuration.

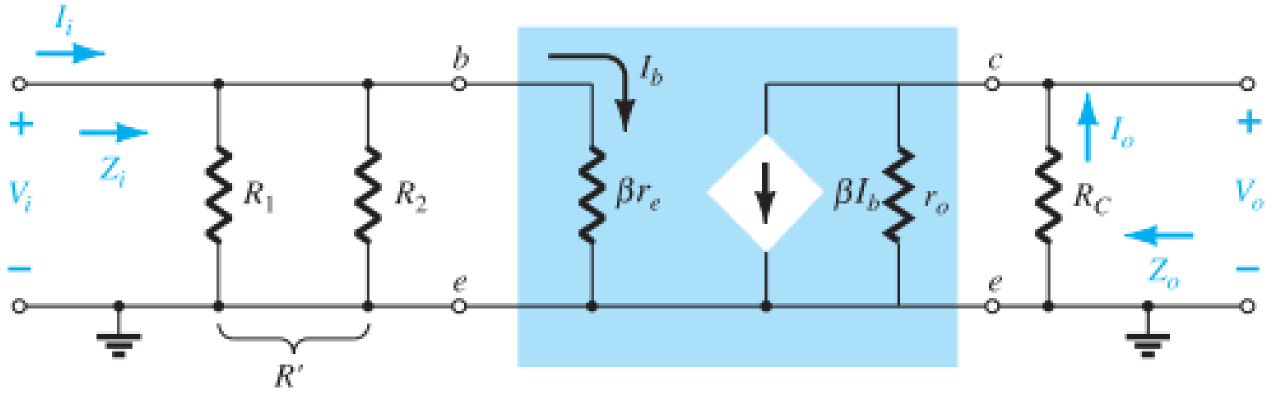


FIG. 5.27
Substituting the r_e equivalent circuit into the ac equivalent network of Fig. 5.26.

$$R' = R_1 \| R_2 = \frac{R_1 R_2}{R_1 + R_2}$$

$$Z_i = R' \| \beta r_e$$

$$Z_o = R_C \| r_o$$

$$Z_o \cong R_C \quad {}_{r_o \geq 10R_C}$$

$$V_o = -(\beta I_b)(R_C \| r_o)$$

$$I_b = \frac{V_i}{\beta r_e}$$

$$V_o = -\beta\left(\frac{V_i}{\beta r_e}\right)(R_C \| r_o)$$

$$A_v = \frac{V_o}{V_i} = \frac{-R_C \| r_o}{r_e}$$

$$A_v = \frac{V_o}{V_i} \cong -\frac{R_C}{r_e} \quad {}_{r_o \geq 10R_C}$$

180° phase shift

C.E. Emitter Bias Configuration (Un-bypassed without ro)

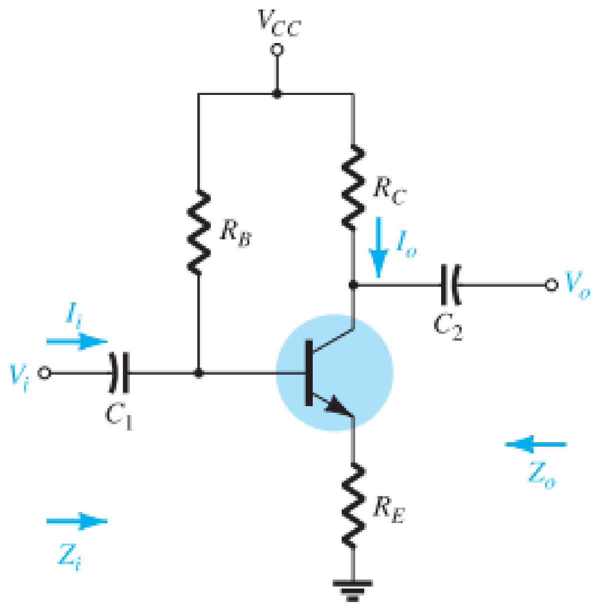


FIG. 5.29
CE emitter-bias configuration.

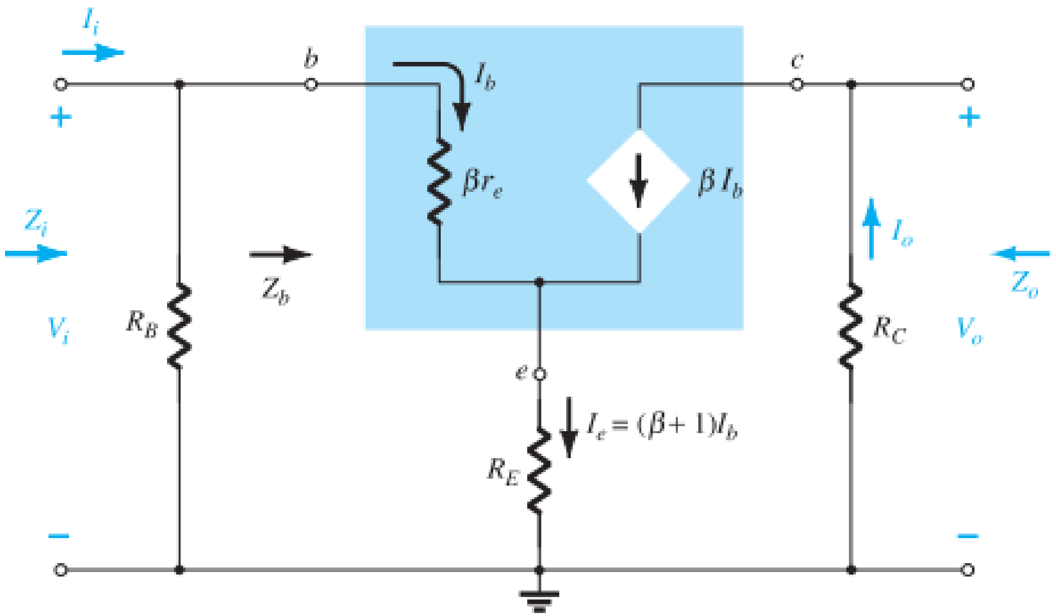


FIG. 5.30
Substituting the r_e equivalent circuit into the ac equivalent network of Fig. 5.29.

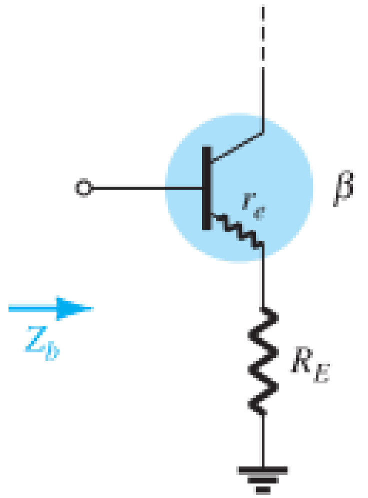


FIG. 5.31
Defining the input impedance of a transistor with an unbypassed emitter resistor.

$$V_i = I_b\beta r_e + I_e R_E$$

$$V_i = I_b\beta r_e + (\beta + 1)I_b R_E$$

$$Z_b = \frac{V_i}{I_b} = \beta r_e + (\beta + 1)R_E$$

$$\boxed{Z_b = \beta r_e + (\beta + 1)R_E}$$

$$Z_b \cong \beta r_e + \beta R_E$$

$$\boxed{Z_b \cong \beta(r_e + R_E)}$$

$$\boxed{Z_b \cong \beta R_E}$$

$$\boxed{Z_i = R_B \| Z_b}$$

$$\boxed{Z_o = R_C}$$

C.E. Emitter Bias Configuration (Un-bypassed without ro)

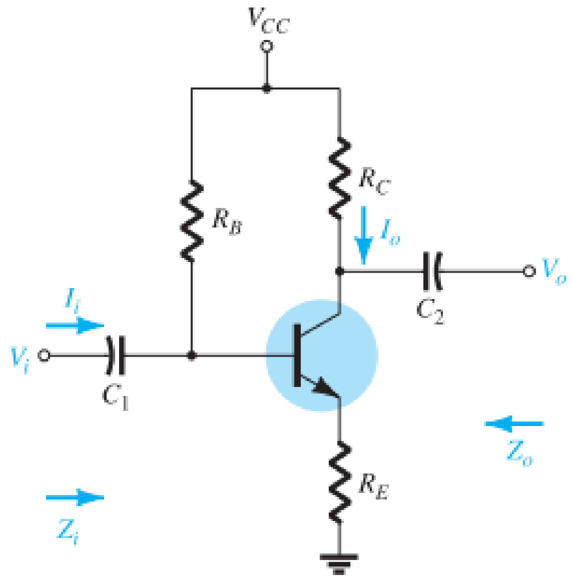


FIG. 5.29
CE emitter-bias configuration.

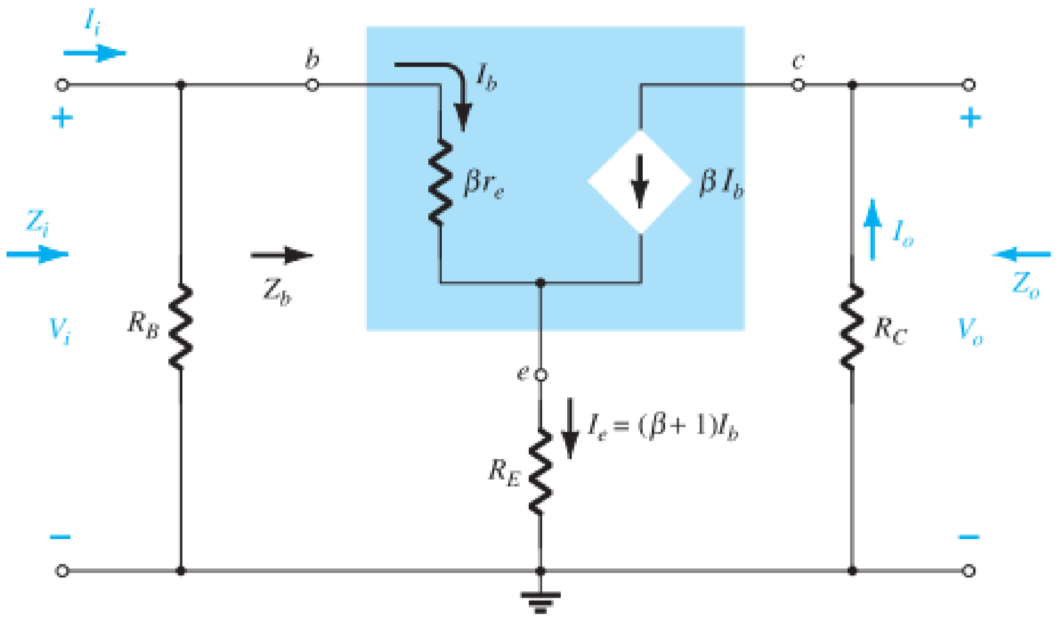


FIG. 5.30
Substituting the r_e equivalent circuit into the ac equivalent network of Fig. 5.29.

$$I_b = \frac{V_i}{Z_b}$$

$$V_o = -I_o R_C = -\beta I_b R_C$$

$$= -\beta\left(\frac{V_i}{Z_b}\right) R_C$$

$$A_v = \frac{V_o}{V_i} = -\frac{\beta R_C}{Z_b}$$

$$Z_b \cong \beta(r_e + R_E)$$

$$A_v = \frac{V_o}{V_i} \cong -\frac{R_C}{r_e + R_E}$$

$$Z_b \cong \beta R_E$$

$$A_v = \frac{V_o}{V_i} \cong -\frac{R_C}{R_E}$$

180° phase shift

C.E. Emitter Bias Configuration (Un-bypassed with ro)

$$Z_i = R_B \| Z_b$$

$$Z_b = \beta r_e + \left[\frac{(\beta + 1) + R_C/r_o}{1 + (R_C + R_E)/r_o}\right] R_E$$

R_C/r_o is always much less than $(\beta + 1)$,

$$Z_b \cong \beta r_e + \frac{(\beta + 1)R_E}{1 + (R_C + R_E)/r_o}$$

For $r_o \geq 10(R_C + R_E)$,

$$Z_b \cong \beta r_e + (\beta + 1)R_E$$

$$Z_b \cong \beta(r_e + R_E)$$

$r_o \geq 10(R_C + R_E)$

$$Z_o = R_C \| \left[r_o + \frac{\beta(r_o + r_e)}{1 + \frac{\beta r_e}{R_E}} \right]$$

$r_o \gg r_e$,

$$Z_o \cong R_C \| r_o \left[1 + \frac{\beta}{1 + \frac{\beta r_e}{R_E}} \right]$$

$$Z_o \cong R_C \| r_o \left[1 + \frac{1}{\frac{1}{\beta} + \frac{r_e}{R_E}} \right]$$

Typically $1/\beta$ and r_e/R_E are less than one with a sum usually less than one.

$$Z_o \cong R_C$$

Any level of r_o

$$A_v = \frac{V_o}{V_i} = \frac{-\frac{\beta R_C}{Z_b}\left[1 + \frac{r_e}{r_o}\right] + \frac{R_C}{r_o}}{1 + \frac{R_C}{r_o}}$$

$\frac{r_e}{r_o} \ll 1$,

$$A_v = \frac{V_o}{V_i} \cong \frac{-\frac{\beta R_C}{Z_b} + \frac{R_C}{r_o}}{1 + \frac{R_C}{r_o}}$$

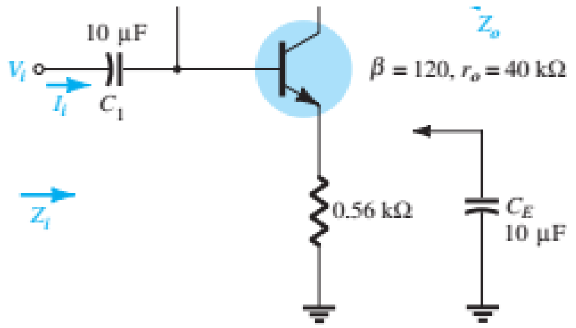
$r_o \geq 10R_C$,

$$A_v = \frac{V_o}{V_i} \cong -\frac{\beta R_C}{Z_b}$$

$r_o \geq 10R_C$

C.E. Emitter Bias Configuration (bypassed)

Same as CE fixed bias config.



Portion bypassed

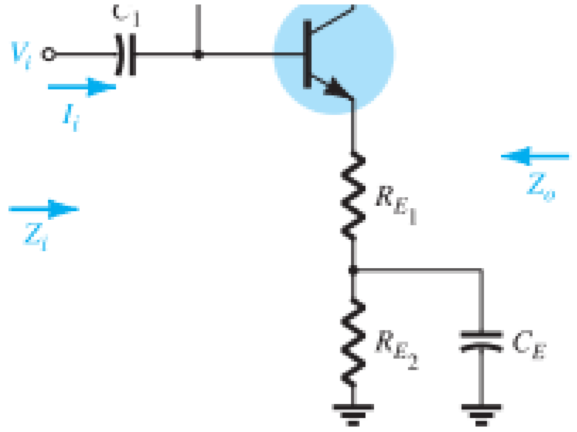


FIG. 5.35
An emitter-bias configuration with a portion of the emitter-bias resistance bypassed in the ac domain.

C.E. Emitter Bias Configuration (Example)

EXAMPLE 5.3 For the network of Fig. 5.32, without C_E (unbypassed), determine:

a. r_e.
b. Z_i.
c. Z_o.
d. A_v.

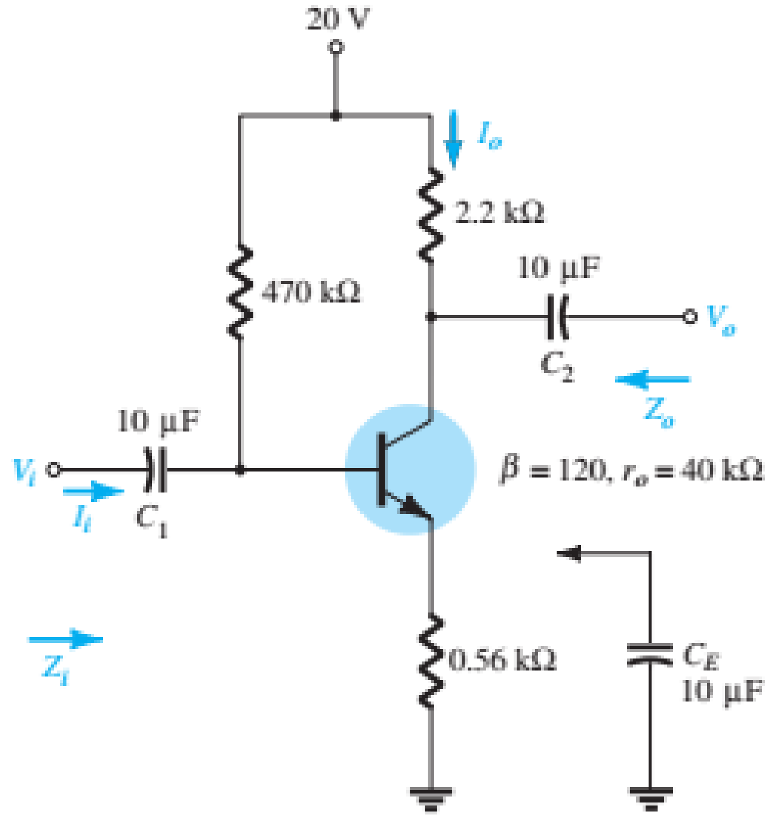


FIG. 5.32
Example 5.3.

Solution:

a. DC:

$$I_B = \frac{V_{CC} - V_{BE}}{R_B + (\beta + 1)R_E} = \frac{20\text{ V} - 0.7\text{ V}}{470\text{ k}\Omega + (121)0.56\text{ k}\Omega} = 35.89\ \mu\text{A}$$

$$I_E = (\beta + 1)I_B = (121)(35.89\ \mu\text{A}) = 4.34\text{ mA}$$

and
$$r_e = \frac{26\text{ mV}}{I_E} = \frac{26\text{ mV}}{4.34\text{ mA}} = \mathbf{5.99\ \Omega}$$

b. Testing the condition $r_o \geq 10(R_C + R_E)$, we obtain

$$40\text{ k}\Omega \geq 10(2.2\text{ k}\Omega + 0.56\text{ k}\Omega)$$
$$40\text{ k}\Omega \geq 10(2.76\text{ k}\Omega) = 27.6\text{ k}\Omega\ (\textit{satisfied})$$

Therefore,

$$Z_b \cong \beta(r_e + R_E) = 120(5.99\ \Omega + 560\ \Omega) = 67.92\text{ k}\Omega$$

and

$$Z_i = R_B \| Z_b = 470\text{ k}\Omega \| 67.92\text{ k}\Omega = \mathbf{59.34\text{ k}\Omega}$$

c. $Z_o = R_C = \mathbf{2.2\text{ k}\Omega}$

d. $r_o \geq 10R_C$ is satisfied. Therefore,

$$A_v = \frac{V_o}{V_i} \cong -\frac{\beta R_C}{Z_b} = -\frac{(120)(2.2\text{ k}\Omega)}{67.92\text{ k}\Omega} = \mathbf{-3.89}$$

compared to −3.93 using Eq. (5.20): $A_v \cong -R_C/R_E$.

C.E. Emitter Bias Configuration (Example)

EXAMPLE 5.3 For the network of Fig. 5.32, without C_E (unbypassed), determine:

a. r_e.
b. Z_i.
c. Z_o.
d. A_v.

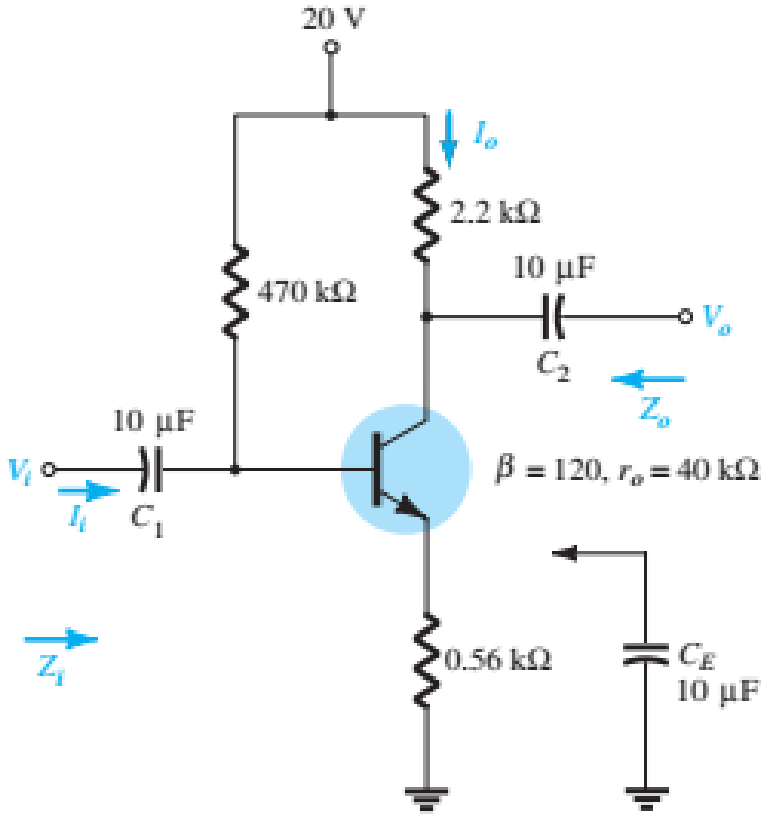


FIG. 5.32
Example 5.3.

EXAMPLE 5.4 Repeat the analysis of Example 5.3 with C_E in place.

Solution:

a. The dc analysis is the same, and $r_e = 5.99\ \Omega$.
b. R_E is "shorted out" by C_E for the ac analysis. Therefore,

$$Z_i = R_B \| Z_b = R_B \| \beta r_e = 470\ \text{k}\Omega \| (120)(5.99\ \Omega)$$
$$= 470\ \text{k}\Omega \| 718.8\ \Omega \cong \mathbf{717.70\ \Omega}$$

c. $Z_o = R_C = \mathbf{2.2\ k\Omega}$

d. $A_v = -\dfrac{R_C}{r_e}$

$$= -\frac{2.2\ \text{k}\Omega}{5.99\ \Omega} = \mathbf{-367.28}\ \text{(a significant increase)}$$

Emitter Follower (Common Collector) Configuration

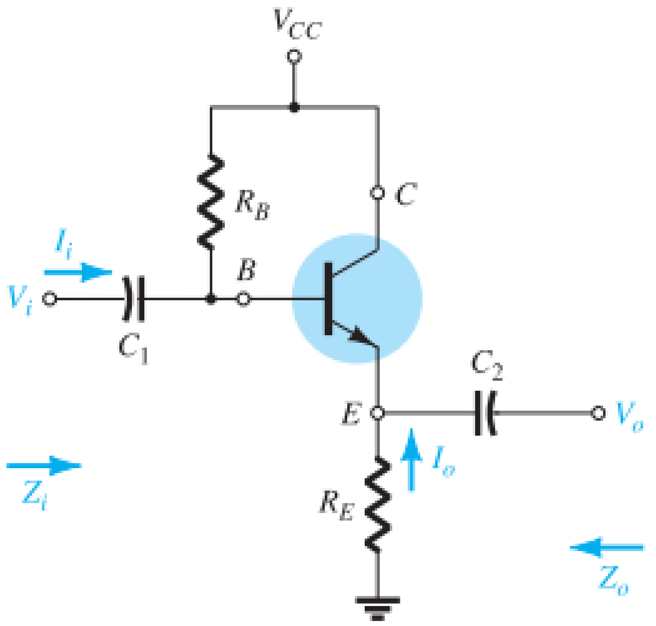


FIG. 5.36
Emitter-follower configuration.

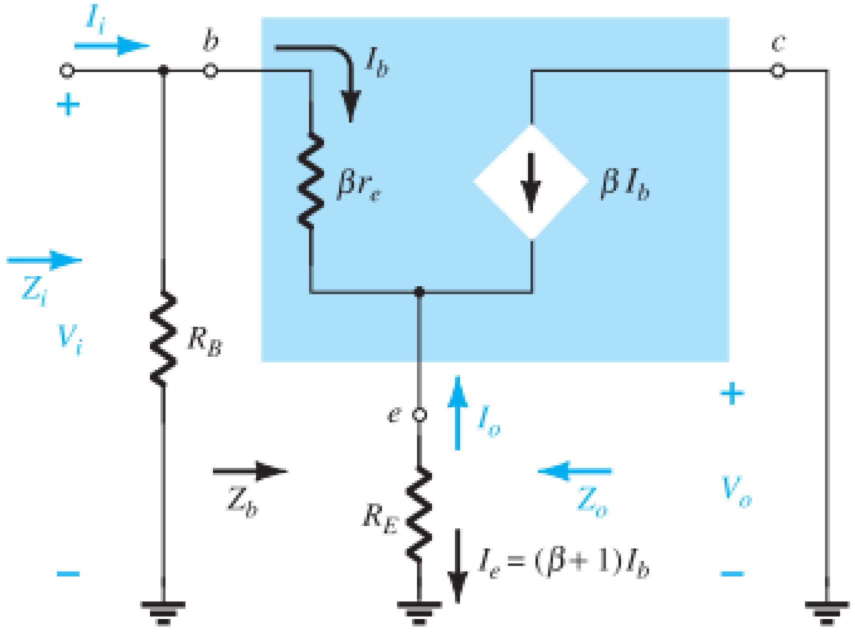


FIG. 5.37
Substituting the r_e equivalent circuit into the ac equivalent network of Fig. 5.36.

$$Z_i = R_B \| Z_b$$

$$Z_b = \beta r_e + (\beta + 1)R_E$$

$$Z_b \cong \beta(r_e + R_E)$$

$$Z_b \cong \beta R_E \qquad R_E \gg r_e$$

Emitter Follower Configuration (o/p impedance and Gain)

$$I_e = (\beta + 1)I_b = (\beta + 1)\frac{V_i}{Z_b}$$

$$I_e = \frac{(\beta + 1)V_i}{\beta r_e + (\beta + 1)R_E}$$

$$I_e = \frac{V_i}{[\beta r_e/(\beta + 1)] + R_E}$$

$$(\beta + 1) \cong \beta$$

$$\frac{\beta r_e}{\beta + 1} \cong \frac{\beta r_e}{\beta} = r_e$$

$$I_e \cong \frac{V_i}{r_e + R_E}$$

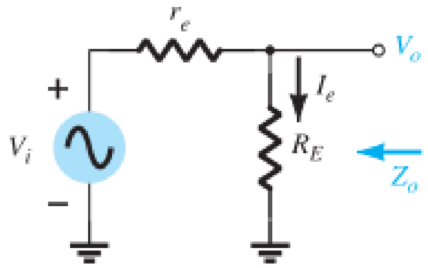


FIG. 5.38
Defining the output impedance for the emitter-follower configuration.

$$Z_o = R_E \| r_e$$

$$Z_o \cong r_e$$

$$V_o = \frac{R_E V_i}{R_E + r_e}$$

$$A_v = \frac{V_o}{V_i} = \frac{R_E}{R_E + r_e}$$

Because R_E is usually much greater than r_e,

$$R_E + r_e \cong R_E;$$

$$A_v = \frac{V_o}{V_i} \cong 1$$ in phase

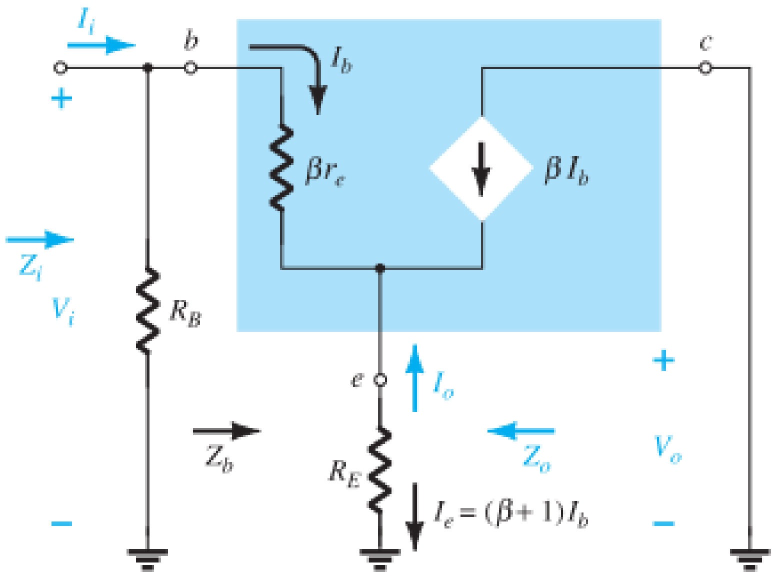


FIG. 5.37
Substituting the r_e equivalent circuit into the ac equivalent network of Fig. 5.36.

Emitter Follower Configuration..

Effect of r_o

$$Z_b = \beta r_e + \frac{(\beta + 1)R_E}{1 + \frac{R_E}{r_o}}$$

$$r_o \geq 10R_E$$

$$Z_b = \beta r_e + (\beta + 1)R_E$$

$$Z_b \cong \beta(r_e + R_E)$$

$r_o \geq 10R_E$

$$Z_o = r_o \| R_E \| \frac{\beta r_e}{(\beta + 1)}$$

$$Z_o = r_o \| R_E \| r_e$$

$$Z_o \cong R_E \| r_e$$

Any r_o

$$Z_o \cong r_e$$

$$A_v = \frac{(\beta + 1)R_E/Z_b}{1 + \frac{R_E}{r_o}}$$

$$A_v \cong \frac{\beta R_E}{Z_b}$$

$$Z_b \cong \beta(r_e + R_E)$$

$$A_v \cong \frac{\beta R_E}{\beta(r_e + R_E)}$$

$$A_v \cong \frac{R_E}{r_e + R_E}$$

$r_o \geq 10R_E$

Emitter Follower Configuration (Example (wo/ro)

EXAMPLE 5.7 For the emitter-follower network of Fig. 5.39, determine:

a. r_e.
b. Z_i.
c. Z_o.
d. A_v.
e. Repeat parts (b) through (d) with $r_o = 25\ \text{k}\Omega$ and compare results.

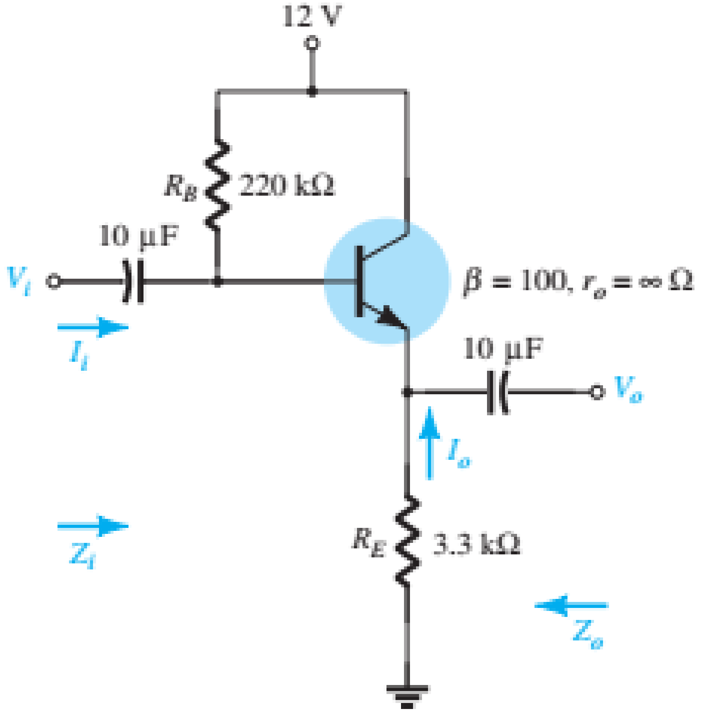


FIG. 5.39

Solution:

a. $$I_B = \frac{V_{CC} - V_{BE}}{R_B + (\beta + 1)R_E}$$
$$= \frac{12\ \text{V} - 0.7\ \text{V}}{220\ \text{k}\Omega + (101)3.3\ \text{k}\Omega} = 20.42\ \mu\text{A}$$
$$I_E = (\beta + 1)I_B$$
$$= (101)(20.42\ \mu\text{A}) = 2.062\ \text{mA}$$
$$r_e = \frac{26\ \text{mV}}{I_E} = \frac{26\ \text{mV}}{2.062\ \text{mA}} = \mathbf{12.61\ \Omega}$$

b. $$Z_b = \beta r_e + (\beta + 1)R_E$$
$$= (100)(12.61\ \Omega) + (101)(3.3\ \text{k}\Omega)$$
$$= 1.261\ \text{k}\Omega + 333.3\ \text{k}\Omega$$
$$= 334.56\ \text{k}\Omega \cong \beta R_E$$
$$Z_i = R_B \| Z_b = 220\ \text{k}\Omega \| 334.56\ \text{k}\Omega$$
$$= \mathbf{132.72\ k\Omega}$$

c. $$Z_o = R_E \| r_e = 3.3\ \text{k}\Omega \| 12.61\ \Omega$$
$$= \mathbf{12.56\ \Omega} \cong r_e$$

d. $$A_v = \frac{V_o}{V_i} = \frac{R_E}{R_E + r_e} = \frac{3.3\ \text{k}\Omega}{3.3\ \text{k}\Omega + 12.61\ \Omega}$$
$$= \mathbf{0.996} \cong \mathbf{1}$$

Emitter Follower Configuration (Example w/ro)

e. Checking the condition $r_o \geq 10R_E$, we have

$$25\text{ k}\Omega \geq 10(3.3\text{ k}\Omega) = 33\text{ k}\Omega$$

which is *not* satisfied. Therefore,

$$Z_b = \beta r_e + \frac{(\beta + 1)R_E}{1 + \frac{R_E}{r_o}} = (100)(12.61\ \Omega) + \frac{(100 + 1)3.3\text{ k}\Omega}{1 + \frac{3.3\text{ k}\Omega}{25\text{ k}\Omega}}$$

$$= 1.261\text{ k}\Omega + 294.43\text{ k}\Omega$$

$$= 295.7\text{ k}\Omega$$

with $Z_i = R_B \| Z_b = 220\text{ k}\Omega \| 295.7\text{ k}\Omega$

$= \mathbf{126.15\text{ k}\Omega}$ vs. $132.72\text{ k}\Omega$ obtained earlier

$Z_o = R_E \| r_e = \mathbf{12.56\ \Omega}$ as obtained earlier

$$A_v = \frac{(\beta + 1)R_E/Z_b}{\left[1 + \frac{R_E}{r_o}\right]} = \frac{(100 + 1)(3.3\text{ k}\Omega)/295.7\text{ k}\Omega}{\left[1 + \frac{3.3\text{ k}\Omega}{25\text{ k}\Omega}\right]}$$

$$= \mathbf{0.996 \cong 1}$$

matching the earlier result.

Emitter Follower Configuration (Diff. Biasing)

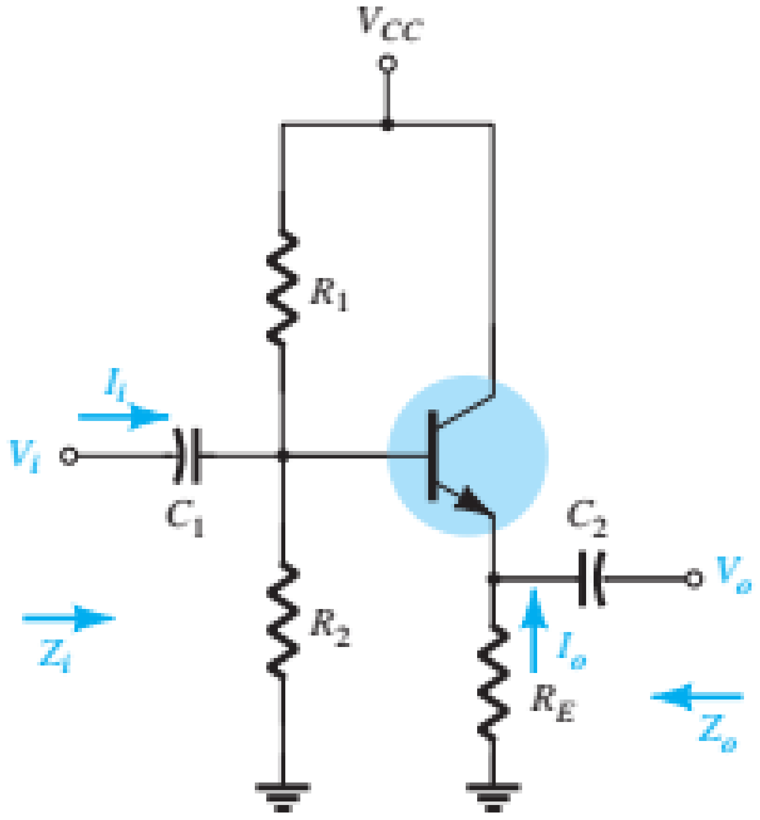


FIG. 5.40

Emitter-follower configuration with a voltage-divider biasing arrangement.

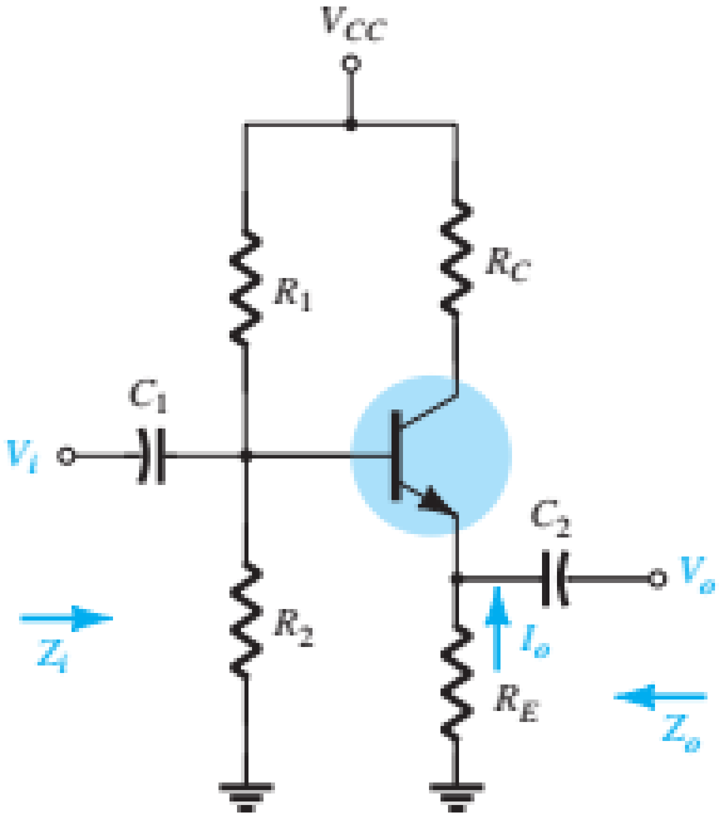


FIG. 5.41

Emitter-follower configuration with a collector resistor R_C.


Thank You!